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VISUALIZATION OF BUBBLE–WAKE INTERACTIONS FOR A STREAM OF BUBBLES IN A TWO-DIMENSIONAL LIQUID–SOLID FLUIDIZED BED

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Abstract—In-line bubble–bubble interactions were studied visually for a stream of bubbles injected from a single nozzle into a two-dimensional liquid–solid fluidized bed. The bubble size varied from 12 to 35 mm in breadth, with corresponding bubble Re ranging from 2000 to 13000. Coalescence may take place due to suction of the trailing bubble into the primary wake of the leading bubble driven by the pressure defect in this region. Breakup probably occurs when the trailing bubble roof is flattened due to free shear layer penetration and/or vortical flow in the near wake of the leading bubble. The structure of the "apparent" primary wake for bubble aggregates in close contact varies depending on the geometric configuration of the aggregates. The wake shedding frequency was obtained for both colliding and non-colliding bubbles. For a given bubble Re, the St was found to be within the same range as that for single-bubble conditions for non-colliding bubbles but to be lower for colliding bubbles.

Key Words: bubble–wake interaction, liquid–solid, fluidized bed, bubble collision, coalescence, breakup, wake interference, vortex shedding

INTRODUCTION

In gas–liquid/gas–liquid–solid contacting devices such as bubble column and three-phase fluidized bed reactors, bubble coalescence and breakup play a crucial role in determining the distributions of bubble size and rise velocity and gas–liquid interfacial area. Bubble coalescence in liquids has been studied both experimentally and theoretically for two successive bubbles (Crabtree & Bridgwater 1971; Narayanan *et al.* 1974; Bhaga & Weber 1980; Komasawa *et al.* 1980), in a chain of bubbles (de Nevers & Wu 1971) and in a swarm of bubbles (Otake *et al.* 1977). Based on the observations made in most of these studies, coalescence of a pair of bubbles with stable toroidal wakes proceeds via the following steps: (1) vertical aligning of both bubbles along the same axis; (2) acceleration and elongation of the trailing bubble; (3) the trailing bubble overtaking the leading one; and (4) drainage and rupture of the thin film of liquid separating the two bubbles. The presence of the wake of the leading bubble is claimed to be responsible for the bubble–bubble interactions (steps 1–3). The theoretical treatment has been confined to predicting the position or the rise velocity of the trailing bubble during the acceleration period for the case of stable wakes.

Studies of the breakup of a large single gas bubble in liquids and/or liquid–solid fluidized beds (e.g. Clift & Grace 1972; Henriksen & Ostergaard 1974; Grace *et al.* 1978) suggest that inherent bubble splitting in the absence of constant external disturbances is due to Rayleigh–Taylor instability along the bubble frontal surface. When such disturbances are present, however, the bubble is more easily disintegrated. For instance, shear stresses present in the liquid flow can break the bubble long before the instability grows (Buckmaster 1973). In three-phase fluidized beds, it has been observed that large, heavy particles will penetrate through the bubble (Henriksen & Ostergaard 1974) and often result in bubble breakage (Lee 1965; Ostergaard 1969; Bruce & Revel-Chion 1974; Lee *et al.* 1974). The mechanism of bubble breakup caused by these external disturbances is quite different from that caused by Rayleigh–Taylor instability. Under strong bubble–bubble interactions, the mechanism due to external disturbances is considered to predominate.

Direct observations of the bubble–bubble interactions in a swarm of bubbles, which prevail in practical gas–liquid/gas–liquid–solid contactors, have not been fully conducted. Otake *et al.* (1977)

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visually studied the bubble coalescence and breakup phenomena in a swarm of bubbles in liquids and presented experimental evidence that both phenomena do take place, even if the bubble size is not very large [i.e. much smaller than the maximum stable sizes predicted by Grace *et al.* (1978)]. Otake *et al.* (1977) identified a criterion which determines whether coalescence or breakup occurs, based on the interaction between two successive bubbles and the trajectories of the trailing bubble. By their criterion bubble coalescence takes place when more than about half the cross-sectional area of the trailing bubble overlaps that of the leading bubble at a critical distance, while bubble breakup occurs if the overlapping is less than about half the cross-sectional area of the trailing bubble. At the critical distance the leading bubble begins to exert a noticeable influence on the trailing one. They concluded that the wake of the leading bubble is responsible for both coalescence of the two bubbles and breakup of the trailing bubble.

The wake flow characteristics can be phenomenologically described based on the dynamic/cyclic nature of large-scale vortical motion for both non-turbulent and phase-averaged turbulent wakes (Perry *et al.* 1982). The periodicity of the wake is best represented by the frequency of vortex shedding. Vortex shedding from solid bluff bodies has been studied for many decades; e.g. for single cylinders by Fage & Johansen (1927), Roshko (1955), Sarpkaya (1975), Gerrard (1978), Cantwell & Coles (1983) and Nakagawa (1987) and for arrangements of multiple cylinders by Zdravkovich (1977), Williamson (1985), Hayashi *et al.* (1986) and Hetz *et al.* (1988).

There exists a definite relationship between the Strouhal number, St, or the reduced shedding frequency and the Reynolds number, Re, for each geometry of the bodies in a certain Re range. St is known to be almost insensitive to Re (based on the maximum transverse dimension) for single bodies in the Re range 10^3–10^5. As the number of bodies increases, however, the wake structure becomes less coherent and the phenomenon may not be characterized by a discrete frequency. Hetz *et al.* (1988) explained the flow over five circular cylinders arranged in line in terms of the interaction of three basic oscillating modes: (1) cavity oscillations; (2) gap shedding; and (3) bluff–body shedding. Hetz *et al.* found the first to be insignificant in magnitude compared to the others or to "lock on" to the dominant shedding mode. The flow behind cylinders arranged side by side is also complex (Williamson 1985; Hayashi *et al.* 1986). A variety of flow patterns, depending on the size of the gap between the bodies, are observed, such as vortex-shedding synchronization either in phase or in antiphase, formation of a "binary-vortex" street, which comprises pairs of like-signed vortices rotating around one another, and harmonic modes of vortex shedding, i.e. the shedding frequency on one side of the wake being a multiple of that on the other.

For gas bubbles, however, the St vs Re relationship is known to be different from that for fixed bodies, due to bubble oscillation or rocking under the influence of asymmetric vortex shedding. The energy associated with the wake-shedding process for gas bubbles can be exchanged between the bubble and the surrounding medium, while all the shedding energy for the fixed solid body is confined to the surrounding fluid alone. The bubbles also change their shapes and relative locations during the interaction between the bubbles and their wakes. Bubble coalescence and breakup play salient roles in bubble–wake dynamics.

The objectives of the present study are to examine the bubble coalescence and breakup phenomena in a stream of bubbles, to elucidate the effect of the wake of the leading bubble on these phenomena and to study the variation of the wake structure due to interacting bubbles in the presence of solid particles. Direct visualization of the bubble and wake flow is performed through a two-dimensional bed.

EXPERIMENTAL

A schematic diagram of the experimental system is given in figure 1. A two-dimensional Plexiglas column of 1.0 m height, 0.41 m width and 8 mm nominal gap thickness was used to visually observe the behavior of bubbles rising in line and their wakes. The two-dimensional system was used due mainly to the presence of high concentrations of non-transparent solid particles in the system. Inevitably, there exist wall effects exerted by the two side plates on the flow around the bubble; some liquid and solid particles flow between the walls and the bubble. The bubble exhibits inherent three-dimensional characteristics, especially around the bubble frontal region (e.g. Collins 1965).

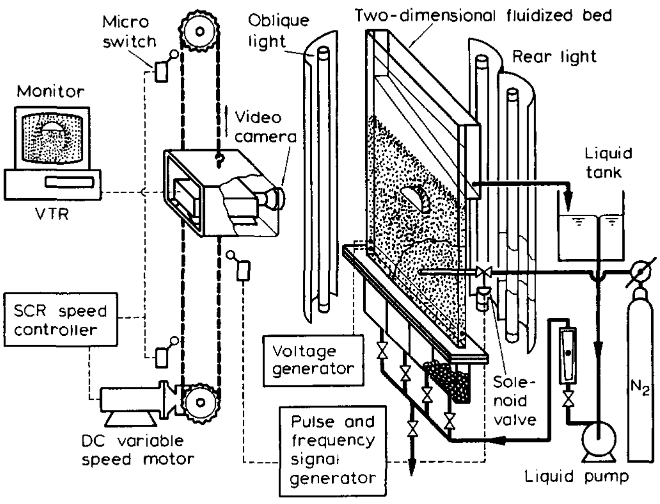


Figure 1. Schematic diagram of the experimental system.

Certain flow characteristics, however, such as the motion of vortices as large coherent structures are still two-dimensional.

A video camera capable of moving vertically at adjustable speeds was employed to monitor the bubble wake behavior in the frame of reference fixed with respect to the subject bubble. A stream of gas bubbles of equal size and spacing were injected through a 6.4 mm o.d. nozzle flush-mounted on the rear wall. The bubble size was readily controlled by altering the gas delivery pressure and/or the opening time of a solenoid valve. The initial spacing between the successive bubbles was varied by altering the pulse frequency generated by the electric signal generator. Details of the vertical bubble tracking system are given in Tsuchiya & Fan (1986).

Nitrogen bubbles were injected into water–solid particle suspension systems. The particles used were all spherical with size and density ranging from 0.46 to 1.5 mm and from 1250 to 2500 kg/m^3, respectively. Detailed particle properties are given in table 1. The bed expansion ratio was kept in the range 1.4–2.0. The corresponding bed voidage ranged from 0.57 to 0.70. The average bed height was maintained between 0.55 and 0.66 m, yielding an in-bed observation zone height of about 0.5 m and a freeboard zone of about 0.3 m. Bubble frequencies (f_b) over the range 1–3 s^{-1} were employed to investigate the bubble–bubble and bubble–wake interactions.

RESULTS AND DISCUSSION

Rise characteristics of bubbles in a stream

The bubble size in the present study ranges from 12 to 35 mm in breadth (major diameter). Over this size range, Tsuchiya & Fan (1986) found that a single bubble travels rectilinearly with rocking or base oscillations. Bubbles in a stream, however, rise in a zig-zag path even before they undergo appreciable bubble–bubble interactions. The present results indicate that the zig-zagging behavior is appreciable if the bubble injection frequency is greater than about 1.5 s^{-1}. Photographs of typical

Table 1. Particle properties and bed voidage

Bed material	Average dia (mm)	Density (kg/m^3)	Terminal velocity (10^{-2} m/s)	Bed voidage
Glass beads (GB460)	0.460	2500	6.73	0.593
Glass beads (GB774)	0.774	2500	11.9	0.581
Activated carbon (AC778)	0.778	1509[a]	5.69	0.698
Acetate balls (AT1500)	1.50	1252	6.99	0.572

[a]Wet density; dry density is 910 kg/m^3.

bubble rise paths observed in water-fluidized beds of 460 and 774 μm glass beads and 1.5 mm acetate particles are presented in figure 2. As shown in the figure, the bubble sheds alternately a series of vortices whose central region appears as a bright spot, establishing a staggered snake-like liquid flow pattern downstream relative to the bubble. This staggered liquid stream from the leading bubble enhances the zig-zag motion of the trailing bubble regardless of particle properties.

When the distance between the adjacent bubbles relative to the breadth of the leading bubble, or relative bubble spacing, L_b/b, is sufficiently small, the trailing bubbles accelerate and eventually collide with the leading bubbles. Figure 3 demonstrates a bubble-pairing process for two different cases. In the first case shown in figure 3(a), three bubbles initially rise with equal bubble spacing in a water–460 μm glass bead fluidized bed. As time proceeds, the first and second bubbles are paired with the second bubble being profoundly elongated. These two bubbles eventually collide. Figure 3(b) shows a typical "catching-up" process observed in a water–1.5 mm acetate particle fluidized bed. The sequence of photographs clearly demonstrates that the bubble pairing (first pair) results in collision of the pair in a manner somewhat similar to the bubble coalescence sequence for a pair of bubbles with laminar wakes described in the introduction. Figure 3(b) presents the entire catching-up process by two pairs of bubbles. Note that the bubble spacing between the paired leading bubbles is almost constant throughout the pairing process regardless of the local acceleration of the paired trailing bubbles.

The shape and rise velocity of a pair of bubbles were measured at several bubble frequencies. Although the shape of the paired leading bubble undergoes much less deformation than that of the paired trailing bubble, the paired leading bubble still exhibits a trend towards vertical elongation before the actual bubble collision takes place. The aspect ratio (height/breadth) of the leading bubble, h/b, increased, on average, by about 21 and 33% for non-colliding and colliding bubbles, respectively. Colliding bubbles refers to bubbles colliding with neighboring bubbles before reaching the upper surface of the fluidized bed. For non-colliding bubbles, it was observed that the rise velocity of the leading bubble does not differ from that of a single bubble (in isolation) of identical size. For colliding bubbles, on the other hand, the rise velocity increased by 22%, on average, over that of a corresponding single bubble of identical size. No systematic variation with respect to particle properties was observed.

Acceleration of the trailing bubble

For bubbles with unsteady turbulent wakes, acceleration of the trailing bubble towards the leading bubble is limited by distance (Komasawa *et al.* 1980). A measure of this limit of the acceleration effect is the maximum bubble spacing which results in eventual collision of these bubbles. This maximum bubble spacing is hereafter referred to as the critical bubble spacing. Figure 4 shows the variation of L_b/b, which was obtained from the average bubble rise velocity and bubble frequency, with the bubble Reynolds number, Re_b $(=bU_b/\nu)$. Here U_b is the bubble rise velocity relative to the surrounding liquid and ν is the liquid kinematic viscosity. In the figure, the critical bubble spacing is seen to be around 5–7 times the bubble breadth and relatively insensitive to Re_b. For comparison, the corresponding distance reported by other investigators for bubbles in three-dimensional liquid media is 5–6 times the equivalent bubble diameter for two successive bubbles with non-laminar wakes (Komasawa *et al.* 1980) and 3–4 times the equivalent bubble diameter for two bubbles in a swarm of bubbles (Otake *et al.* 1977).

Acceleration of the trailing bubble towards a leading bubble of equal or greater size due to the presence of the wake of the leading bubble has been indicated by several researchers (Crabtree & Bridgwater 1971; de Nevers & Wu 1971; Narayanan *et al.* 1974; Otake *et al.* 1977; Bhaga & Weber 1980; Komasawa *et al.* 1980). Although gradual acceleration starts at 5–7 times the bubble width below the leading bubble base, rapid acceleration accompanied by profound shape elongation of the trailing bubble in the vertical direction occurs only within the "primary wake." The primary wake was identified and defined by Tsuchiya & Fan (1986) as the region inherently associated with an individual bubble. The primary wake is fluid mechanically distinct from the far wake in the sense that it is responsible for the formation of vortices and is relatively insensitive to external flow conditions. In the following, local interactions of bubbles in the primary wake are presented.

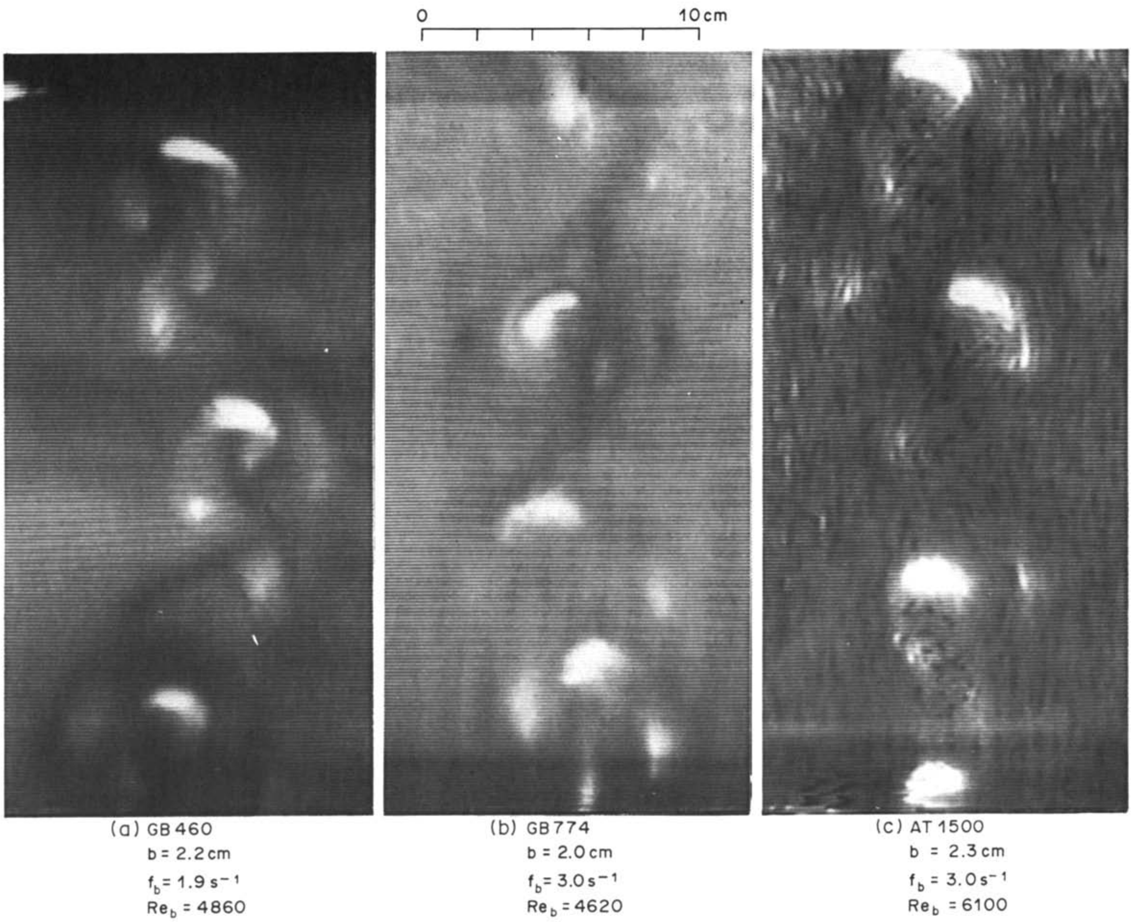
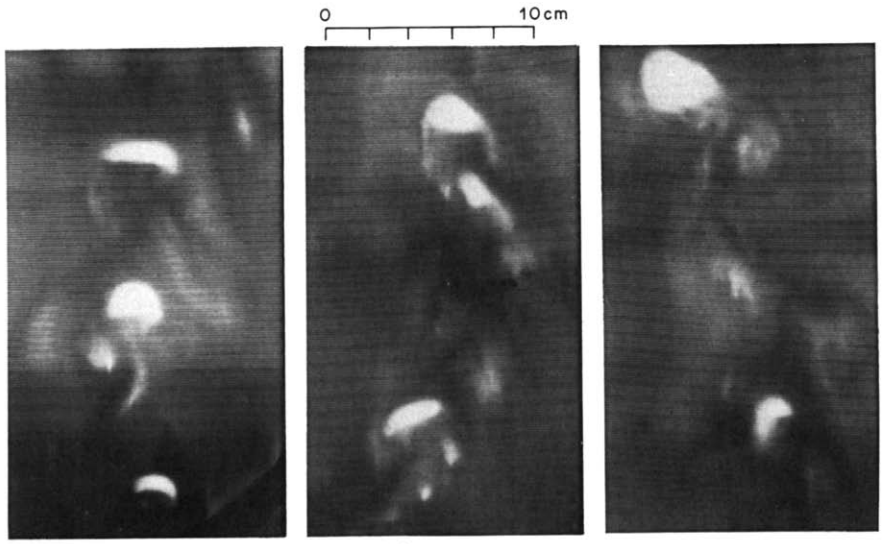


Figure 2. Effect of vortex shedding on the bubble rise path.

Coalescence/breakup mechanisms due to bubble–wake interactions

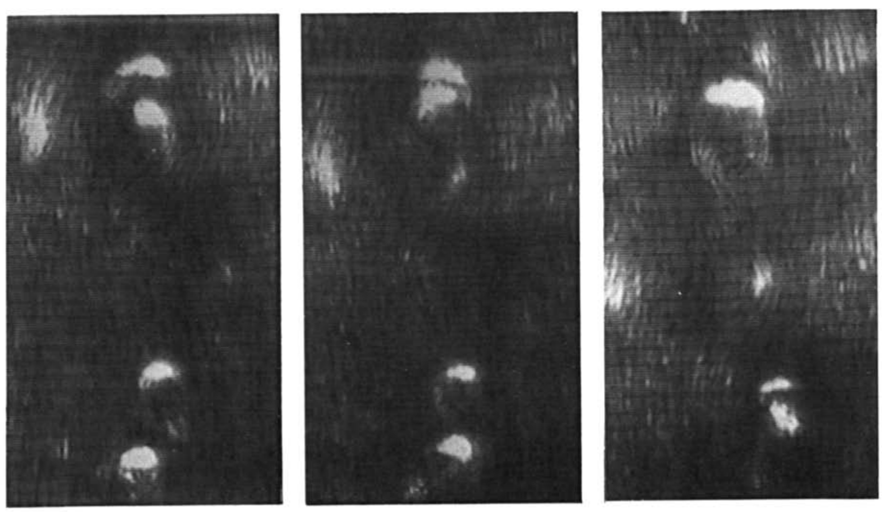
The location, shape deformation, coalescence and breakup of a pair of bubbles in a stream of bubbles were monitored after the trailing bubble (labeled bubble 2) moved to the near wake of the leading bubble (labeled bubble 1). Figures 5 and 6 show, respectively, a series of photographs reflecting the bubble coalescence and breakup phenomena in a water–778 μm activated carbon particle ($\rho_s = 1509$ kg/m^3) fluidized bed. In the photographs, the bubbles appear as bright blobs whose outline is sometimes blurred due to halation or persistence of vision (ghost) from the previous image in the video system. The particle flow around the bubbles is represented by streaks of tracer particles.

The first three photographs in figure 5, spanning a period of 0.1 s, depict a period of rapid acceleration of bubble 2 towards bubble 1. In figure 5(a) bubble 2 is still outside the primary wake of bubble 1. Note an almost enclosed region (marked by the flow of the tracer particles) existing immediately beneath bubble 1 with a vortex (appearing less dark due to lower solids concentration) at the lower bound of this region. As bubble 2 crosses the primary wake boundary [figure 5(b)], it deforms in the direction of its movement and accelerates rapidly. Immediately after bubble 2 moves to the primary wake of bubble 1 [figure 5(c)], the speed of bubble 2 reaches a maximum, as evidenced by an apparent tail or "ghost" behind it. The last three photographs in figure 5 show the final stage of the bubble collision–coalescence process. Bubble 2 first invades the central region of the primary wake of bubble 1 [figure 5(d)]. As the thin film of liquid–solid mixture between the two bubbles [figure 5(e)] drains, they collide and coalescence takes place [figure 5(f)]. Note an enclosed region in figures 5(e, f) whose size (relative to bubble size) is about the same (extending about one bubble breadth downstream of the bubble base) as that in figure 5(a), indicating that the relative primary-wake size is almost invariant throughout the bubble coalescence process. The



(a) GB460

b = 2.8cm, f_b = 3.1 s^{-1}, Re_b = 9180



(b) AT1500

b = 2.0 cm, f_b = 3.0 s^{-1}, Re_b = 4040

Figure 3. Bubble pairing followed by bubble collision.

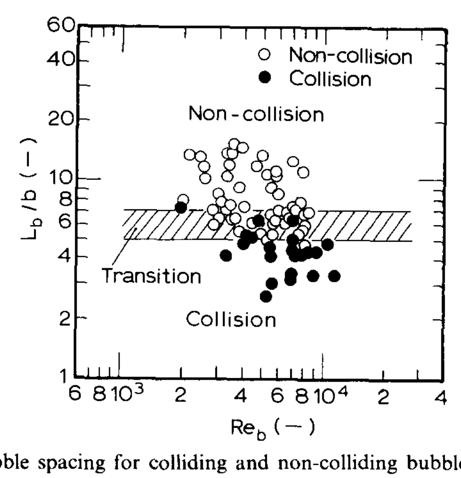


Figure 4. Relative bubble spacing for colliding and non-colliding bubbles with turbulent wakes.

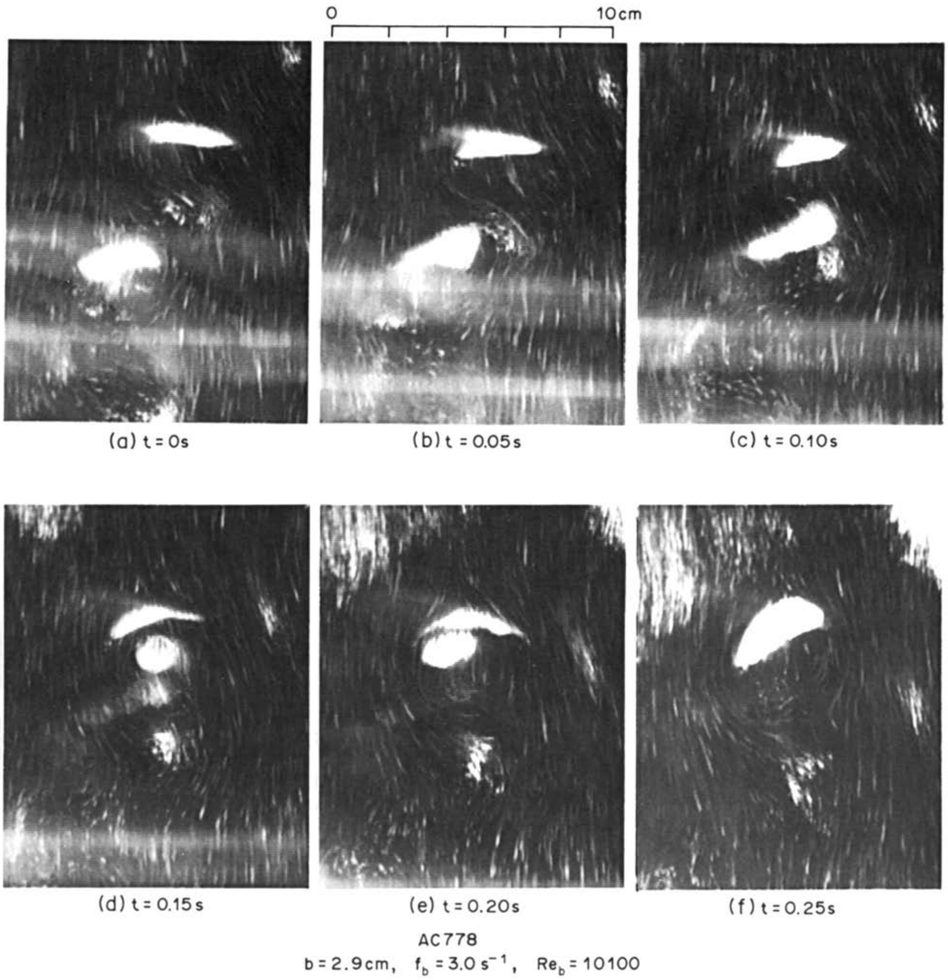
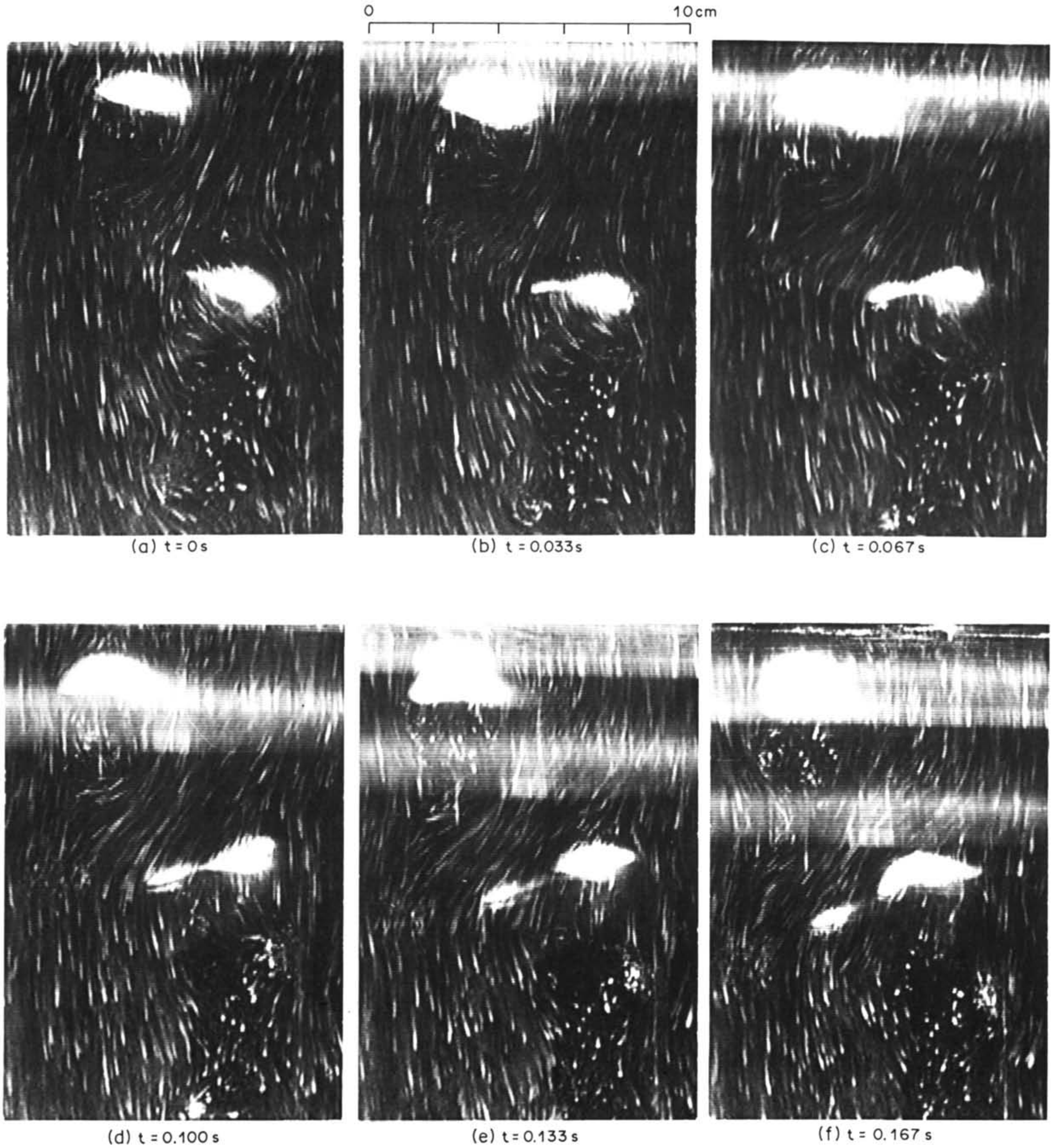


Figure 5. Sequence of bubble coalescence in a water–778 μm activated carbon particle fluidized bed.

time-averaged primary-wake area was reported to be 3.3 $\pm$ 1.2 times the bubble area over the range $1500 < Re_b < 8150$ and insensitive to Re_b as well (Tsuchiya & Fan 1986).

Figure 6 shows the phenomena of splitting of bubble 2 by the shear flow created by bubble 1. In this case, bubble 2 has not reached the primary wake of bubble 1 nor has it overlapped with bubble 1 in the vertical direction [figure 6(a)]. Splitting of bubble 2 is triggered by horizontal entrainment of a part of bubble 2 around the left edge [cf. figure 6(b) with figure 6(a)], followed by flattening [figure 6(b)], indentation [figure 6(c)], necking [figure 6(d)] and eventual breakage [figures 6(e, f)].

More distinctive observations in a time sequence were made for rising bubbles in the freeboard region of a water–1.5 mm acetate particle (ρ_s = 1252 kg/m^3) fluidized bed, shown in figure 7. The time increment in each step (designated by the number in the figure) is 0.0167 s. The first two exposures show bubble 1 erupting from the bed surface with the primary wake. Right after bubble 2 moves to the primary wake of bubble 1 (after step 6), bubble 2 deforms from an oblate shape (flattened horizontally) to a prolate shape (elongated vertically) accompanied by rapid acceleration. Less than 0.15 s elapses before the two bubbles collide. During this rapid acceleration period, the path of bubble 2 is partially influenced by the circulatory flow pattern induced by the vortex in the primary wake of bubble 1. That is, bubble 2 approaches bubble 1 along the wake center axis where there is an upward stream of liquid–solid mixture relative to bubble 1. When bubble 2 reaches the base of bubble 1, bubble 2 is rapidly swept to the side of bubble 1 (left side in the figure) and contacts the edge of bubble 1 via a very thin liquid film (step 16 in the figure). The two bubbles now appear to be united into a single large bubble; however, due possibly to surface-active agents



AC778

$b = 3.0\,\mathrm{cm}$, $f_b = 3.0\,\mathrm{s}^{-1}$, $Re_b = 9960$

Figure 6. Sequence of bubble breakup in a water–778 μm activated carbon particle fluidized bed.

present in the tap water used in this study, they never coalesce. Instead, they begin to rise in parallel with some interactions (step 19).

The remaining part of figure 7 shows that, when two bubbles of almost identical size ascend side by side but in close contact, low pressure created in the wake of bubble 1 can drive part of bubble 2 into the wake. This transverse suction, which occurs rapidly, results in flattening of bubble 2. Once stretched, bubble 2 is susceptible to breakup. The more flattened the bubble roof, the more the interfacial tension force holding the bubble to a single body is weakened. When a large portion of bubble 2 is drawn into the wake immediately beneath the base of bubble 1, bubble 2 breaks into two with the aid of the free shear layer from the edge of bubble 1 (steps 25 and 26). The free shear layer, which tends to roll up towards the wake central axis (Gerrard 1966; Saffman & Baker

1979; Tsuchiya & Fan 1986), can provide sufficient shear to create an indentation on the bubble roof and to withdraw part of the stretched bubble (bubble 2) into the wake of the influencing bubble (bubble 1). This withdrawing–stretching–disintegration process is repeated as long as the bubbles rise side by side in close contact at comparable rise velocities (steps 23–26 and 35–38).

The rate of coalescence upon two-bubble collision was estimated for bubbles of three different size ranges in fluidized beds of four different particles. The results are summarized in table 2. For the bubble size and particle properties considered in the present study, the rate of coalescence was found to be higher for a larger bubble size for a given particle. Comparing the rate of coalescence for GB460 and AT1500 which have about the same terminal velocities, it is shown that the rate of coalescence is higher for particles of smaller size in a fixed bubble size range. Although the

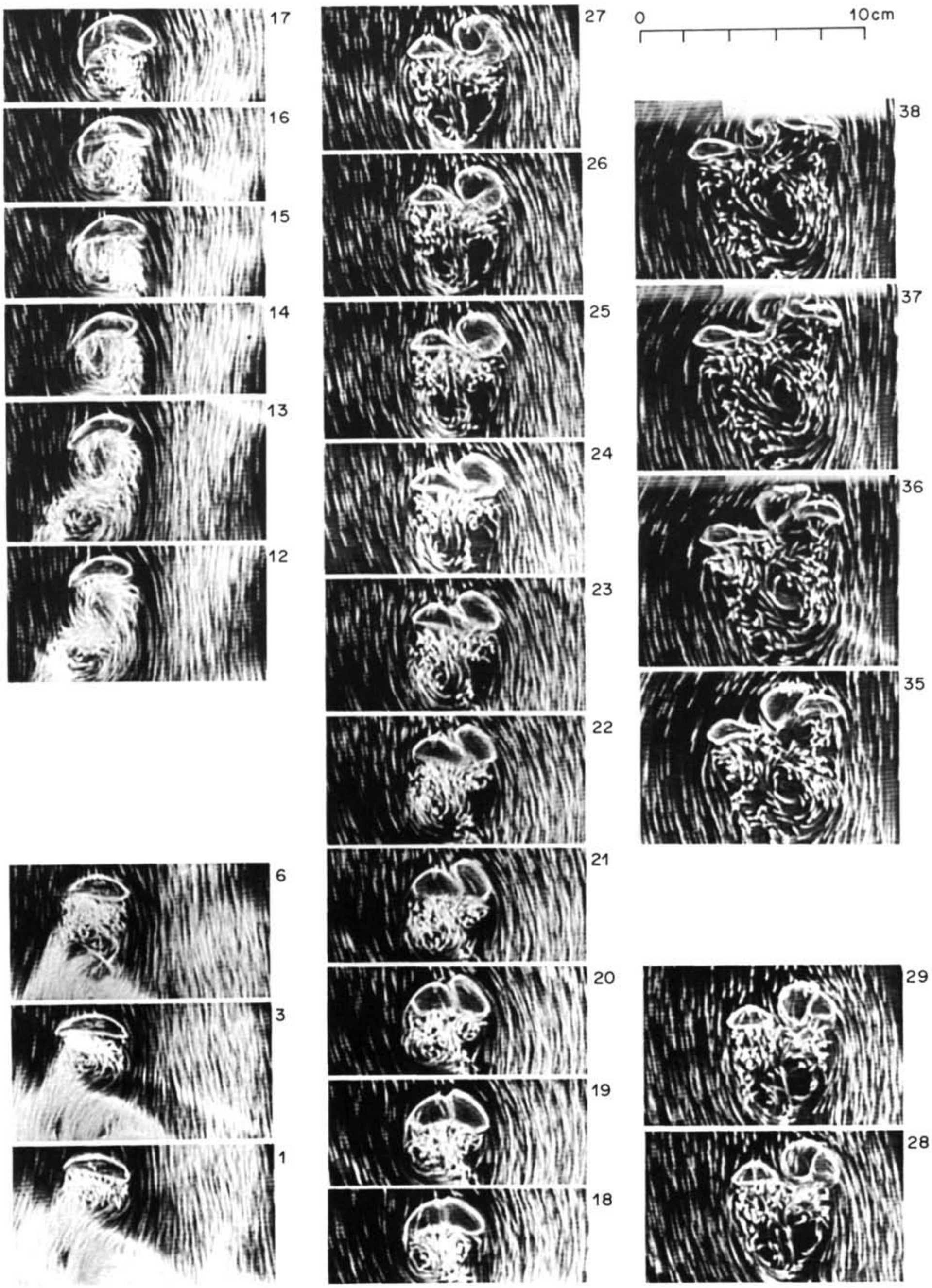


Figure 7. Interaction sequence of bubbles with turbulent wakes in the freeboard of a water–1.5 mm acetate particle fluidized bed.

Table 2. Rate of coalescence (%) upon two-bubble collision

Bed material	Bubble breadth (mm)		
	12–17	20–27	27–35
GB460	40.0	75.0	80.0
GB774	—	—	66.7
AC778	50.0	40.0	83.3
AT1500	12.5	55.6	63.6

present results are not based on a large number of data, the qualitative trends given here reflect the intrinsic behavior of interacting bubbles in a liquid–solid medium.

Wake interference of bubbles rising side by side

While wake flow characteristics play a critical role in bubble–bubble interactions, interactive rising bubbles exhibit wake structure different from that of a single bubble. Figure 8 presents

Figure 8. Wake interference of bubbles rising in a group in the freeboard of a water–1.5 mm acetate particle fluidized bed.

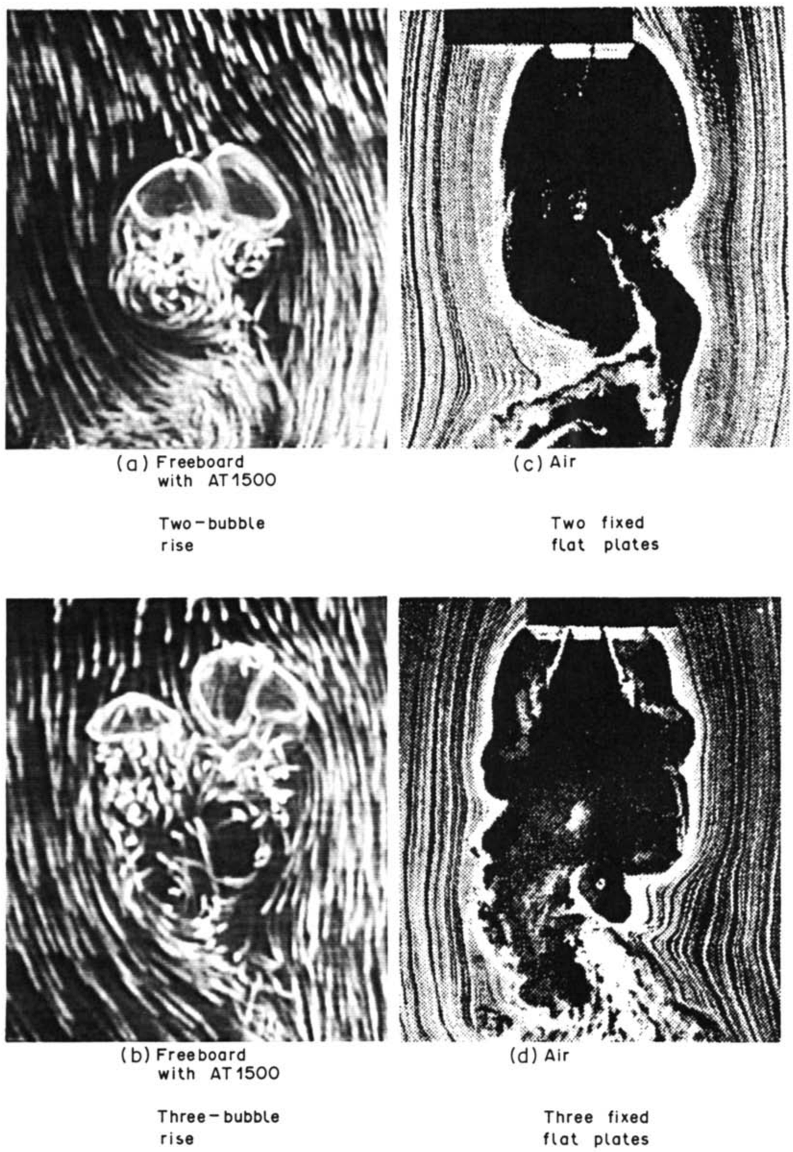


Figure 9. Similarity in wake interference between a horizontal row of bubbles and that of fixed flat plates under a small slit ratio condition. Photographs (c) and (d) from Hayashi *et al.* (1986).

photographs of wake structures at several stages/phases selected from the dynamic events given in figure 7. Wake flow is visualized through the motions of entrained particles. As can be seen in the figure, wake structure as well as the apparent wake size strongly depends on the alignment of rising bubbles, whether vertical or horizontal. When the trailing bubble moves into the primary wake of the leading bubble, the internal structure of the primary wake is disturbed, while the size of the primary wake does not change significantly [figure 8(b)].

When a few bubbles of about the same size ascend side by side in close contact, there appears to be a single large wake beneath a horizontal alignment of the bubbles [figures 8(d, e)]. The size of the resulting "apparent" wake is larger than the sum of the primary-wake size of each constituent bubble when it exists singly, and the internal structure is more complicated than that of the primary wake of a single bubble. This is because the gap(s) between the bubbles is (are) quite narrow and, thus, flow through the gap is not consistently strong.

As an analogy, Hayashi *et al.* (1986) studied wake interference of a row of normal flat plates arranged side by side in a uniform flow of fluid and found that the wake flow behind the plates exhibited very different characteristics for different slit ratios (the ratio of the gap width to the plate

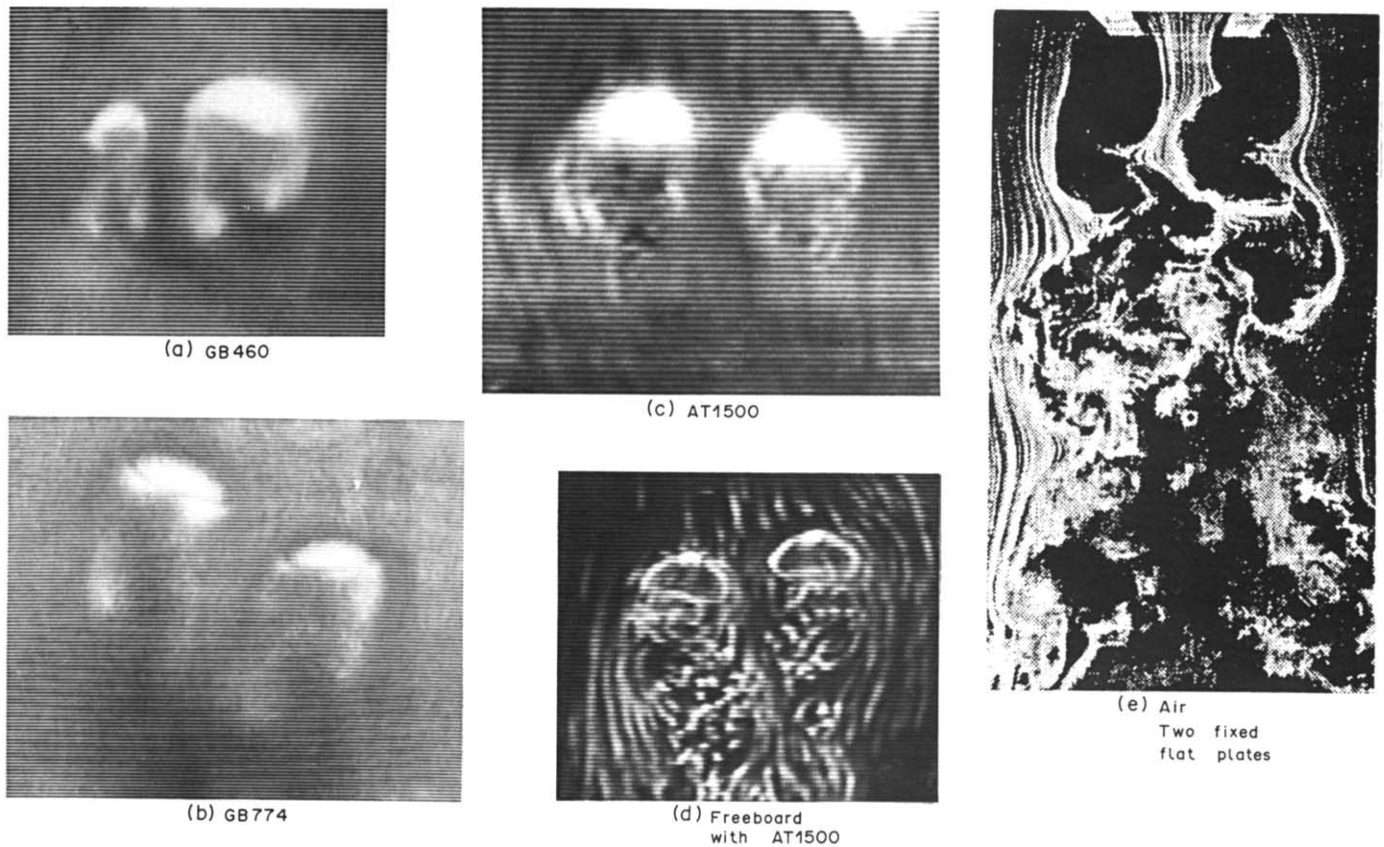


Figure 10. Similarity in wake interference between a horizontal row of bubbles and that of fixed flat plates under a large slit ratio condition. Photograph (e) from Hayashi *et al.* (1986).

width). When the slit ratio was very small (<0.5), the gap flow was weak. The free shear layers separated from the outermost edges of the plate row (consisting of two, three or even four plates) interfered with each other and rolled up to form a very large-scale vortical structure [see figures 9(c, d)]. Hayashi *et al.* claimed that the resulting wake behavior was very similar to the wake behavior behind a single plate. It was not until the slit ratio exceeded 2.0 that individual plates generated their own primary wakes without any significant wake interference from the neighboring plates [see figure 10(e)].

Qualitative similarities in wake interference phenomena thus exist between the horizontal row of bubbles and that of fixed flat plates. Such similarities are demonstrated in figures 9 and 10 for small and large gap or slit ratios, respectively. A separate experiment conducted in this study consisted of injecting two bubbles of identical size simultaneously from two nozzles located at the same height above the liquid distributor. It was found that two bubbles rise side by side without any appreciable interference provided that the horizontal gap between the bubbles is wide enough [see figures 10(a–d)]. If, on the other hand, the gap is not sufficiently large, two or three bubbles create a single, large apparent wake as if they were a single bubble [see figures 9(a, b)]. The bubble configurations depicted in figures 9(a, b) are, however, not stable, resulting in the dynamic bubble–bubble interactions shown in figure 7.

Wake shedding frequency

Figure 11 shows the variation of the shedding frequency of individual vortices, expressed in terms of the bubble Strouhal number, St_b ($=f_v b/U_b$), with Re_b. Here f_v is the measured shedding frequency of individual vortices. Open and solid symbols represent the data obtained in the present study for non-colliding and colliding bubbles, respectively. No appreciable effects of particle properties are seen in either case. The two dotted lines represent the correlated experimental results obtained by Tsuchiya & Fan (1986) for a single bubble in the same two-dimensional fluidized bed for two distinct shedding modes, i.e. alternate and parallel shedding modes. As can be seen in the figure, the vortex-shedding frequency in the absence of bubble collision falls within the same range as that for the alternate vortex-shedding frequency from a single bubble over the entire Re_b range. The parallel shedding was not observed for successive bubbles due to the staggered liquid flow established by the leading bubbles. Possible reasons for the alternate shedding frequency being insensitive to the presence/absence of non-colliding neighboring bubbles are as follows.

1. The frequency of bubble rocking may not be affected by the presence of neighboring bubbles. It can be postulated that the rocking motion of the bubble is independent of the bubble lateral movement (Tsuchiya & Fan 1988), which may be enhanced by external disturbances caused by the neighboring bubbles.

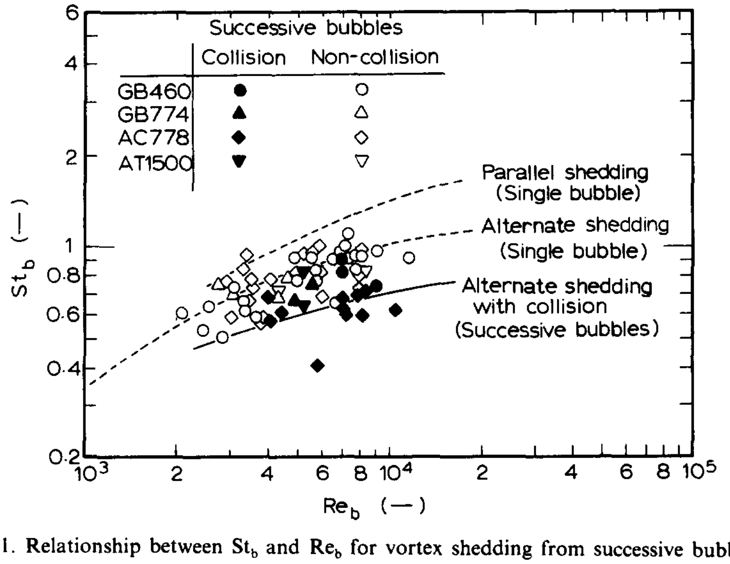


Figure 11. Relationship between St_b and Re_b for vortex shedding from successive bubbles.

2. The frequency of bubble rocking and that of alternate vortex shedding from the bubble can be synchronized in the neighborhood of the natural frequency (i.e. the frequency at which a freely vibrating system oscillates once deflected from the equilibrium position) of the bubble–wake system (Tsuchiya & Fan 1986, 1988). The synchronization, or lock-in, phenomenon is also known to occur for transversely oscillating bluff objects (e.g. Sarpkaya 1979; Williamson & Roshko 1986).

Vortex shedding from the leading bubbles which collide with the trailing ones occurs at somewhat lower frequencies than that from the non-colliding leading bubbles. Figure 11 shows the reduction of St_b by an average of about 30% from the single-bubble case when bubbles collide. The decrease in the shedding frequency itself from the single-bubble case, however, is $\ll 30\%$. Note, St_b is inversely proportional to the bubble rise velocity, which was found to increase for colliding bubbles by about 22%. Still, some effect of bubble collision on the rate of vortex shedding may exist, due probably to the change of the internal structure of the primary wake of the leading bubble. It is speculated that the alternating rhythm of bubble rocking, and hence vortex shedding, established for streaming bubbles is disturbed by the invasion of the trailing bubble into the primary wake of the subject bubble. This invasion may delay the shedding timing.

CONCLUDING REMARKS

The rise of in-line bubbles is characterized by significantly enhanced staggered/zig-zag bubble motion over that of single bubbles. This observation clearly indicates that the rise path and the wake-shedding mode are closely interrelated. Alternate vortex shedding induces a snake-like liquid flow pattern, while the staggered liquid flow enhances the alternating rhythm of vortex shedding. Particle properties have no significant effect on the rise pattern.

Bubble pairing takes place when the bubble spacing becomes less than about 5 times the bubble breadth. The critical bubble spacing ratio is insensitive to Re_b. The mechanism of bubble coalescence is attributed to the vertical suction of the trailing bubble caused by low pressure in the primary wake of the leading bubble. The bubble breakup mechanism, on the other hand, is attributed to horizontal withdrawing, stretching, necking and disintegration of the trailing bubble effected by the turbulent shear and vortical flow near the edges of the leading bubble.

When a few bubbles of comparable size rise as a group, the wake structure is influenced by the bubble alignment. Especially when the bubbles are closely aligned horizontally, there appears to be a single large wake below the aligned bubbles. The apparent increase in the primary-wake size can contribute to increased capacity of the wake for solids mixing. The wake-shedding frequency appears independent of particle properties. It is comparable for both single and in-line bubbles, provided that the primary wake is not interfered with by the trailing bubble. For colliding bubbles, vortex shedding tends to be less rhythmic and the shedding frequency is slightly lower than that for non-colliding bubbles.

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